2018-08-31-1

* First quiz Friday September 7, 2018

Covers HW #1

* Section 2.4 in textbook

Last Time:

$P = VI\cos(\theta_v - \theta_i)$

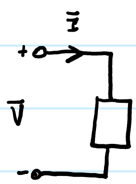
$Q = VI\sin(\theta_v - \theta_i)$

$\bar{S} = \bar{V}\bar{I}^* = VI\angle\theta_v - \theta_i = P + jQ$

$PF = \cos(\theta_v - \theta_i) \Rightarrow PF = \cos(\theta)$

Today:
1) Why do reactances give Q?
2) Power triangle method for calculating power
3) Conservation of complex power
4) Specifying power

Ex 1



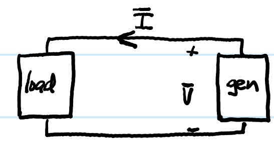
$\bar{V} = 100\angle 10^\circ$

$\bar{I} = 5\angle 130^\circ$

$\bar{S} = \bar{V}\bar{I}^* = 500\angle -120^\circ$ VA

$\bar{S} = -250 - j433.01$ VA

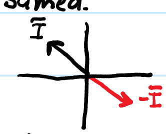
Reality



* P is negative, meaning power is generated, not consumed.

To correct equations, flip direction for defining $\bar{I}$.

* PF only defined for $|\theta| < 90^\circ$

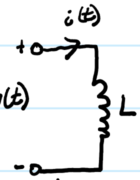


$\bar{I} = -x + jy$ $\quad |\bar{I}| = |-\bar{I}|$

$-\bar{I} = x - jy$ $\quad \theta_i + 180^\circ = \theta_{-I}$

Reactances:

1) Inductor:



$v = L\frac{di}{dt} \Rightarrow v = L\frac{d}{dt}(I_m\cos(\omega t + \theta_i))$

$v = \omega L I_m(-\sin(\omega t + \theta_i))$

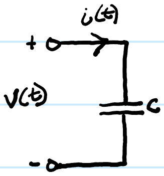
$v = \omega L I_m \sin(\omega t - \theta_i)$

$v = \omega L I_m \cos(-\omega t - \theta_i - 90^\circ)$

$v = \omega L I_m \cos(\omega t + (\theta_i + 90^\circ))$

$\boxed{\bar{V} = \omega L I_{RMS}\angle\theta_i + 90^\circ}$

2) Capacitor



$i(t) = C\frac{dv}{dt} \Rightarrow i = C\frac{d}{dt}(V_m\cos(\omega t + \theta_v))$

$i = \omega C V_m(-\sin(\omega t + \theta_v))$

$i = \omega C V_m \sin(-\omega t - \theta_v)$

$i = \omega C V_m \cos(-\omega t - \theta_v - 90^\circ)$

$i = \omega C V_m \cos(\omega t + (\theta_v + 90^\circ))$

$\boxed{\bar{I} = \omega C V_{RMS}\angle\theta_v + 90^\circ}$

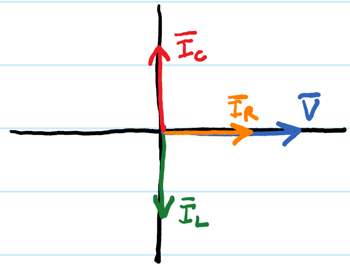
$\bar{S}_L = \bar{V}_L \bar{I}_L^* = \omega L I_{RMS}^2 \angle \theta_i + 90° - \theta_i$

$$\boxed{\begin{array}{l} \bar{S}_L = \omega L I_{RMS}^2 \angle 90° = 0 + j\omega L I_{RMS}^2 \\ PF = 0 \quad \text{lag} \quad (\theta > 0) \end{array}}$$

$\bar{S}_c = \bar{V}_c \bar{I}_c^* = \omega C V_{RMS}^2 \angle \theta_v - (\theta_v + 90°)$

$$\boxed{\begin{array}{l} \bar{S}_c = \omega C V_{RMS}^2 \angle -90° = 0 - j\omega C V_{RMS}^2 \\ PF = 0 \quad \text{lead} \quad (\theta < 0) \end{array}}$$

Plot Phasors in Complex Plane



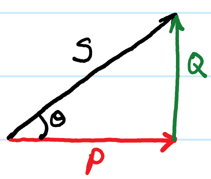
* Resistor is in phase with voltage
* Inductor current lags voltage by 90°
* Capacitor current leads voltage by 90°

Power Triangle:

$|\bar{S}| = S = \sqrt{P^2 + Q^2} = VI$

$P = S\cos(\theta)$

$Q = S\sin(\theta)$



* The diagram above is the power triangle

Ex $\bar{V} = 100\angle 10°$ V

$\bar{I} = 10\angle -26.8°$ A

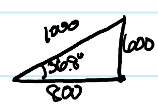
Find: a) S and θ

b) Draw the power triangle to find P and Q

Solution: a) $S = VI$ $\quad \theta = 10° - (-26.8°)$

$$\boxed{S = 1000 \text{ VA} \qquad \theta = 36.8°}$$

b)



$$\boxed{\begin{array}{l} P = 800 \text{ W} \\ Q = 600 \text{ VAR} \end{array}}$$

Ex $\bar{V} = 100\angle 10°$ V

$\bar{I} = 10\angle 46.8°$ A

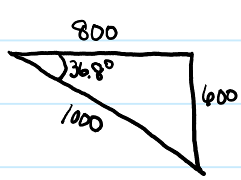
Find: a) S and θ

b) Draw power triangle to find P and Q

Solution: a) $S = VI$ $\quad \theta = \theta_v - \theta_i$

$$\boxed{S = 1000 \text{ VA} \qquad \theta = -36.8°}$$

b)



$$\boxed{\begin{array}{l} P = 800 \text{ W} \\ Q = -600 \text{ VAR} \end{array}}$$

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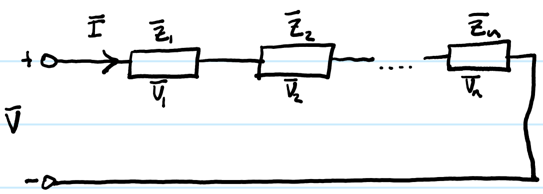
* Up to now, examined a single load. What if there are multiple loads?

Conservation of Complex Power

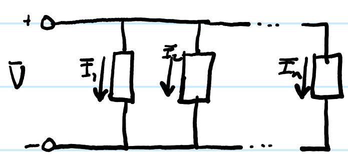
Series:



$\bar{S}_{TOT} = \bar{V}_1\bar{I}^* + \bar{V}_2\bar{I}^* + \ldots + \bar{V}_n\bar{I}^*$

$\boxed{\bar{S}_{TOT} = \bar{S}_1 + \bar{S}_2 + \ldots + \bar{S}_3}$

Parallel:



$\bar{I}_{tot} = \bar{I}_1 + \bar{I}_2 + \ldots \bar{I}_n$

$\bar{S}_{TOT} = \bar{V}\,\bar{I}_{tot}^* = \bar{V}(\bar{I}_1 + \bar{I}_2 + \ldots + \bar{I}_n)^*$

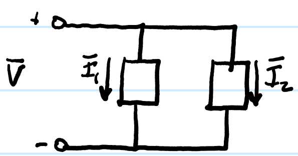
$= \bar{V}\bar{I}_1^* + \bar{V}\bar{I}_2^* + \ldots + \bar{V}\bar{I}_n^*$

$\boxed{\bar{S}_{TOT} = \bar{S}_1 + \bar{S}_2 + \ldots + \bar{S}_n}$

If: $\bar{S}_{TOT} = \sum_{j=1}^{n} \bar{S}_j$, Then: $\boxed{\begin{array}{l} P_{TOT} = P_1 + P_2 + \ldots + P_n \\ Q_{TOT} = Q_1 + Q_2 + \ldots + Q_n \end{array}}$

* We can use this for something called PF correction.

Ex



$\bar{V} = 100\angle 10° \text{ V}$

$\bar{I}_1 = 10\angle -26.8° \text{ A}$

$\bar{I}_2 = 5\angle 40° \text{ A}$

Find: a) $\bar{S}_{tot}$

b) PF_{TOT}

c) Place a capacitor in parallel with the loads. How many VAR's added by capacitor for i) PF = 0.98 lag

ii) PF = 1

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Solution: a) $\bar{S}_{TOT} = \bar{S}_1 + \bar{S}_2$

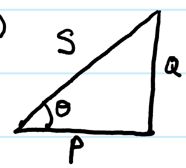
$\bar{S}_1 = V\bar{I}_1^*$ $\qquad$ $\bar{S}_2 = V\bar{I}_2^*$

$\bar{S}_1 = 1000\angle 36.8^\circ$ VA $\qquad$ $\bar{S}_2 = 500\angle -30^\circ$ VA

$= 800 + j600$ VA $\qquad$ $\bar{S}_2 = 433.01 - j250$ VA

$$\boxed{\bar{S}_{TOT} = 1233.01 + j350 \text{ VA}}$$

b)

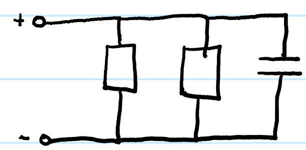


$\tan(\theta) = \frac{Q}{P}$

$\tan^{-1}\left(\frac{Q}{P}\right) = \theta$

$\theta = 15.85^\circ$

$PF = \cos(15.85^\circ) \Rightarrow \boxed{PF = 0.962 \text{ lag}}$



c) Adding capacitor only changes Q

i) $\cos^{-1}(PF_N) = \theta_N$

$\theta_N = 11.5^\circ$

$\tan(\theta_N) = \frac{Q_N}{P} \Rightarrow Q_N = P\tan(\theta_N) \Rightarrow Q_N = 250$ VAR

$Q_0 + Q_c = Q_N$

$Q_c = Q_N - Q_0$

$$\boxed{Q_c = -100 \text{ VAR}}$$

ii) $PF = 1$, $Q = 0$

$$\boxed{Q_c = -350 \text{ VAR}}$$

$PF = 1 \qquad \cos(\theta) = 1 \Rightarrow \theta = 0$

$Q = VI\sin\theta \Rightarrow Q = 0$

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Specifying Power:

6 total quantities: V, I, PF, P, Q, S

* We need a reference. Take: $\bar{V} = V\angle 0$

This gives: $\bar{I} = I\angle\theta$

$$\theta = \begin{cases} \cos(PF) & \text{lag} \\ -\cos(PF) & \text{lead} \end{cases}$$

Different ways of specifying:

1) $\bar{V}$ and $\bar{I}$ given. This is the same as V, I, and PF given

2) V, PF, P given.

$$\theta = \begin{cases} \cos^{-1}(PF) & \text{lag} \\ -\cos^{-1}(PF) & \text{lead} \end{cases} \qquad I = \frac{P}{V\cos\theta} \qquad Q = P\tan\theta$$

$$\bar{S} = P + jQ$$

3) V, PF, S given

$$\theta = \begin{cases} \cos^{-1}(PF) & \text{lag} \\ -\cos^{-1}(PF) & \text{lead} \end{cases} \qquad I = \frac{S}{V} \qquad P = S\cos\theta \qquad \bar{S} = S\angle\theta$$

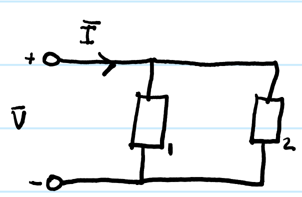
$$Q = S\sin\theta$$

4) V, P, Q given

$$PF = \frac{P}{\sqrt{P^2+Q^2}} \qquad S = P + jQ \qquad \theta = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$I = \frac{\sqrt{P^2+Q^2}}{V}$$

Ex)



$\bar{V} = 1000\angle 0°$

$S_1 = 20$ kVA PF = 0.6 leading

$P_2 = 4$ kW PF = 0.8 lagging

Find: a) $\bar{I}$

b) Input $\bar{S}$

c) Input PF

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Solution: a) V_1, S_1, PF_1 given

$I_1 = \frac{S_1}{V_1} \Rightarrow I_1 = 20A$

$\theta = -\cos^{-1}(0.6) \Rightarrow \theta = -53.13$

$\bar{I}_1 = 20\angle 53.13° \text{ A}$

$\bar{I}_1 = 12 + j16 \text{ A}$

V_2, P_2, PF_2 given

$I_2 = \frac{P_2}{V_2 PF} \Rightarrow I_2 = 5 \text{ A}$

$\theta = \cos^{-1}(0.8) \Rightarrow \theta = 36.87°$

$\bar{I}_2 = 5\angle -36.87°$

$\bar{I}_2 = 4 - j3 \text{ A}$

$\bar{I} = \bar{I}_1 + \bar{I}_2 \Rightarrow \bar{I} = 16 + j13 A$

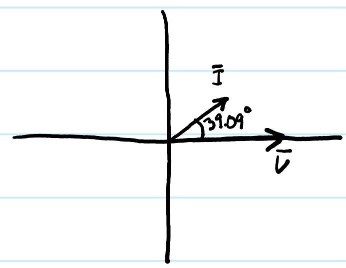
$$\boxed{\bar{I} = 20.62\angle 39.09° \text{ A}}$$

b) $\bar{S} = \bar{V}\bar{I}^*$

$= (1000\angle 0°)(20.62\angle -39.09°)$

$$\boxed{\begin{array}{l}\bar{S} = 20.62\angle -39.09° \text{ kVA} \\ \bar{S} = 16 - j13 \text{ kVA}\end{array}}$$

c)



$PF = \cos(39.09°)$

$$\boxed{PF = 0.776 \text{ leading}}$$